

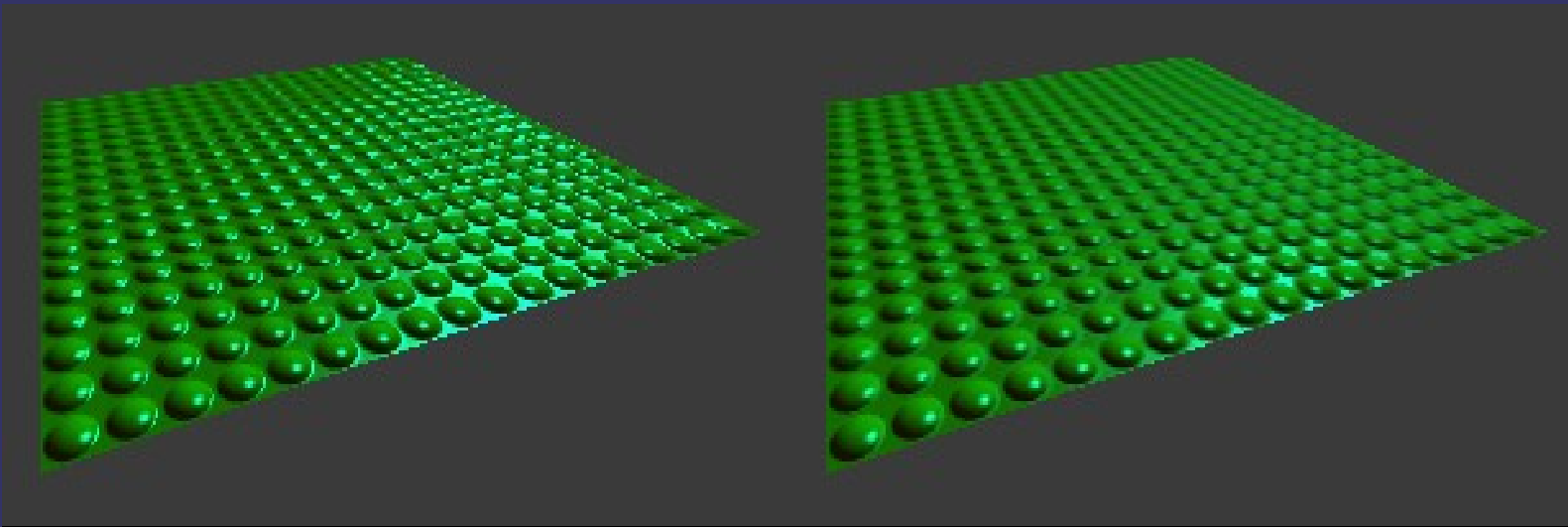
# Computer Graphics Programming II

## ⇒ Agenda:

- Mipmapping normal maps
- Environment map pre-filtering for lighting
- Fresnel reflection
- BRDF introduction
  - Common terminology and notation
  - Cook-Torrance

# Mipmapping Normal Maps

- ⇒ Traditional mipmaps fail for normal maps
  - Averaging multiple vectors yields a vector less than unit length, which leads to lighting artifacts.



Original image from  
[http://developer.nvidia.com/object/mipmapping\\_normal\\_maps.html](http://developer.nvidia.com/object/mipmapping_normal_maps.html)

# Theory

- ⇒ By knowing how much less than unit length the averaged normal is, we can *improve* the lighting
  - Decrease the amount of specular reflection
  - Make the specularity more diffuse-like → decrease the specular exponent

# Toksvig Factor

⇒ Update Blinn's lighting equation:

$$f_t = \frac{|N_a|}{|N_a| + s(1 - |N_a|)}$$
$$K_s = \frac{1 + f_t s}{1 + s} \left( \frac{N_a \cdot H}{|N_a|} \right)^{f_t s}$$

- $f_t$  is the “Toksvig Factor”
- $N_a$  is the averaged normal
- $s$  is the original specular exponent

# *Environment Maps*

- ⇒ Last week we used env. maps for lighting
  - Calculate ideal reflection vector, use that to look up light color from environment map
- ⇒ What is the limitation of this approach and why?

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- ⇒ Last week we used env. maps for lighting
  - Calculate ideal reflection vector, use that to look up light color from environment map
- ⇒ What is the limitation of this approach and why?
  - Really only works for perfectly mirror-like surfaces
    - Surfaces where the specular exponent approaches  $\infty$
  - Only one point of the environment is sampled

# *Env. Map Sampling*

- ⇒ If the problem is that only one point in the original environment map is sampled, what can we do?

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- ⇒ If the problem is that only one point in the original environment map is sampled, what can we do?
  - Obvious answer: take multiple samples
  - Sample  $R$  and a few vectors around  $R$ , and take a weighted average



# *Env. Map Sampling (cont.)*

⇒ What's the problem with this technique?

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- ⇒ What's the problem with this technique?
  - Taking *enough* samples to get good results is slow
  - Taking few enough samples to be fast will give poor looking results
- ⇒ Does this remind you of something?

# *Env. Map Sampling (cont.)*

- ⇒ What's the problem with this technique?
  - Taking *enough* samples to get good results is slow
  - Taking few enough samples to be fast will give poor looking results
- ⇒ Does this remind you of something?
  - Sounds like texture minification
  - To solve that problem we store pre-averaged (filtered) versions of the texture
    - Mipmaps

# *Env. Map Pre-filtering*

⇒ For a cube map:

- Render a single quad for each face
- Each fragment is a unique  $R$  vector
  - Calculate the matching  $V$  vector
- Sample some number of surrounding texels
  - Each texel is an  $L$  vector
  - The contribution of each texel is based on its vector
- Calculate weighted average of all sampled texels, write result

# Env. Map Pre-filtering (cont.)

## ⇒ Notes / caveats:

- This filtered env. map *only* includes the specular component
  - Create a second similar env. map for diffuse
    - This is called an *irradiance map*
  - Can apply essentially *any* lighting model to this technique
- Other env. map types can be handled similarly

*Break*

# *Fresnel Reflection*

- ⇒ Named after French physicist Augustin-Jean Fresnel
  - Pronounced *fray-NELL*
- ⇒ Light moves at different speeds through different materials
  - The ratio of the speed of light in a vacuum to the speed in a particular material is the *refractive index* of that material
  - Glass has an index of refraction of about 1.5

# *Fresnel Reflection*

- ⇒ When light passes through a material with one refractive index to a material with another, several things happen:
  1. The light changes velocity
  2. The light changes direction
    - Since light behaves like a wave, #1 causes #2.
  3. Some of the light is reflected
  4. The remaining light is refracted
    - That is, passes into the material



# *Fresnel and Materials*

- ⇒ Dielectric materials (i.e., things that do *not* conduct electricity) exhibit a strong Fresnel factor
  - Plastic, glass, autopaint, etc.
  - Metal *does not*
  - This fact will be important later...

# *Reflection vs. Refraction*

- ⇒ The amount of reflection depends on the angle between the light and the normal
  - The larger the angle between the normal and the light, the more light is reflected
  - The effect is like a rock skipping on water

# Reflection Math

⇒ The amount of reflection,  $R(\theta)$ , is:

$$c = n_i/n_t (\cos \theta)$$
$$g = \sqrt{1 + c^2 - (n_i/n_t)^2}$$
$$R(\theta) = \frac{1}{2} \left( \frac{g-c}{g+c} \right)^2 \left( 1 + \left( \frac{c(g+c) - (n_i/n_t)^2}{c(g-c) + (n_i/n_t)^2} \right)^2 \right)$$

- $n_i$  is the refractive index of the first material
- $n_t$  is the refractive index of the second material
- $\theta$  is the angle between the surface normal and the light vector

# Reflection Approximation

- ⇒ That math is complex *and* expensive
- ⇒ A good approximation exists:

$$R_a(\theta) = R(0) + (1 - R(0))(1 - \cos(\theta))^5$$

- $R(0)$  is calculated in the application and passed into the shader as a uniform

# *How is it used?*

⇒ Simulate a diffuse surface with a shiny coating:

$$K = (1 - F) K_d + F K_s$$

- The Fresnel term determines what part of the light is reflected by the specular coating
- Whatever light is left is reflected by the diffuse part

# *References*

<http://www.graphics.cornell.edu/~westin/misc/fresnel.html>

[http://physics-animations.com/Physics/English/rays\\_txt.htm](http://physics-animations.com/Physics/English/rays_txt.htm)

[http://en.wikipedia.org/wiki/Fresnel\\_equations](http://en.wikipedia.org/wiki/Fresnel_equations)

*Break*

# BRDFs

## ⇒ Bi-direction reflection distribution function

- Notation is  $f(\omega_o, \omega_i)$

“...describes the ratio of reflected radiance exiting from a surface in a particular direction (defined by the vector  $\omega_o$ ) to the irradiance incident on the surface from direction  $\omega_i$  over a particular waveband.”



# *BRDFs In Lighting*

⇒ The amount of light reflected for a particular angle is:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) (N \cdot \omega_i)$$

- $L(\omega_o)$  is the outgoing light
- $L(\omega_i)$  is the incoming light
- $N$  is the surface normal

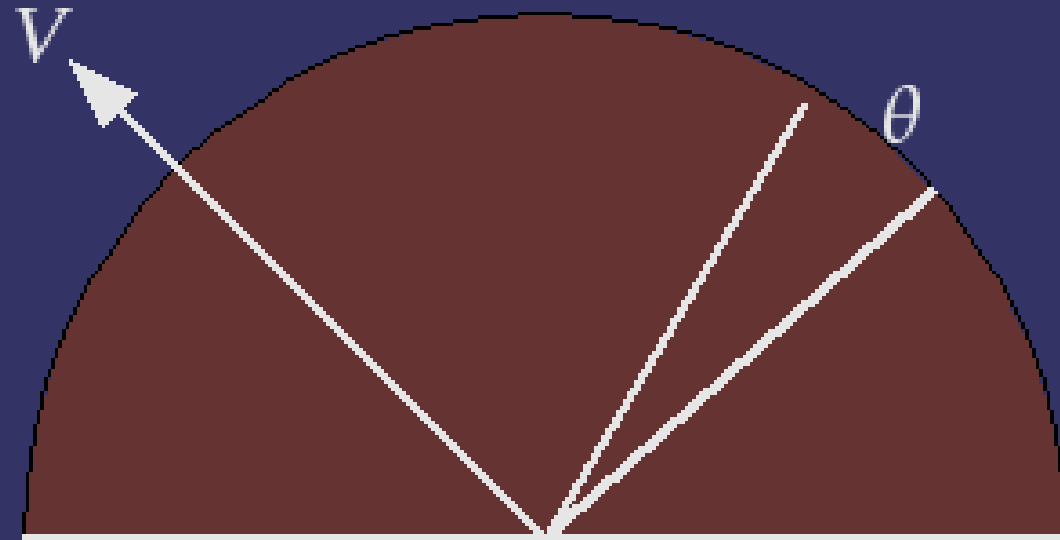
# Notes About Notation

- ⇒  $\omega$  (omega) is used for vectors
  - This is the notation used for BRDFs in physics literature, and all papers about BRDFs use it as well
- ⇒ Exchange  $\omega_i$  and  $\omega_o$  and get the same result
  - This is called Helmholtz Stereopsis or Helmholtz reciprocity
  - It's the “bi-directional” part of BRDF
- ⇒ Measured as a solid angle

# *Solid Angle?*

⇒ Each measurement is flux times solid angle

- To get total light output, integrate over the entire hemisphere



- Results in an extra scale by the total angle,  $\pi$ 
  - Pretty much all BRDFs have a  $1/\pi$  factor to account for this

# *Where does $f$ come from?*

## ⇒ Several common sources:

- Complex camera and light setups can be used to sample a BRDF
  - Oregon BRDF Library (and other) have a lot of this raw data available
- Analytic methods can be used to derive a BRDF
  - We'll cover several common ones this term
  - Most include a Fresnel term

# *Cook-Torrance*

- ⇒ One of the oldest BRDFs used in graphics
  - Developed while Robert Cook was at Lucasfilm and Ken Torrance was at Cornell
  - Published in 1982
- ⇒ Based on “microfacets”

# *Microfacet Primer*

- ⇒ Surfaces are comprised of numerous infinitesimal subsurfaces that act as perfect mirrors
  - Distribution of the subsurface normals determines how specular a surface is
  - Microfacets can obscure other microfacets from both the light and from the viewer
- ⇒ We'll cover both these elements in *much* more detail next week

# Cook-Torrance BRDF

$$f(\omega_o, \omega_i) = K_d f_d + K_s f_s(\omega_o, \omega_i)$$

$$f_d = 1/\pi$$

$$f_s = 1/\pi \frac{F \times D(N \cdot H) \times G(N \cdot \omega_i, N \cdot H, N \cdot \omega_o)}{(N \cdot \omega_i)(N \cdot \omega_o)}$$

# Microfacet Distribution

⇒  $D$  represents the distribution of microfacet normals

- Several models exist
- Cook-Torrance uses the Beckmann Distribution

$$D(N \cdot H) = \frac{1}{4 m^2 (N \cdot H)^4} e^{-\left(\frac{1 - (N \cdot H)^2}{(N \cdot H)^2 m^2}\right)}$$

$m$  is a constant on  $[0, 1]$  that controls the smoothness of the surface



# Geometry Factor

- ⇒ Represents facet-facet occlusion of light source and / or viewer

$$G(N \cdot \omega_i, N \cdot H, N \cdot \omega_o) = \min \left( 1, \frac{2(N \cdot H)(N \cdot \omega_o)}{\omega \cdot H}, \frac{2(N \cdot H)(N \cdot \omega_i)}{\omega \cdot H} \right)$$

- ⇒ Why aren't there subscripts on  $\omega$  in the denominators?
  - Hint: remember  $\omega_i$  is the light and  $\omega_o$  is the viewer

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- ⇒ Why aren't there subscripts on  $\omega$  in the denominators?
  - Hint: remember  $\omega_i$  is the light and  $\omega_o$  is the viewer
  - $H$  is half-way between  $\omega_i$  and  $\omega_o$ , so  $(H \cdot \omega_i) = (H \cdot \omega_o)$

# *Cook-Torrance Diffuse Factor*

⇒ Cook-Torrance diffuse factor:

$$f_d = 1/\pi$$

⇒ “Typical” diffuse factor:

$$K_d = N \cdot L$$

# Cook-Torrance Diffuse Factor

⇒ Cook-Torrance diffuse factor:

$$f_d = 1/\pi$$

⇒ “Typical” diffuse factor:

$$K_d = N \cdot L$$

⇒ Remember how the BRDF equation is used:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) (N \cdot \omega_i)$$

- We just want to scale the incoming energy by the total angle and let the built-in  $(N \cdot \omega_i)$  do the rest
- Remember  $\omega_i$  is  $L$

# References

## ⇒ Measured BRDF data:

<http://math.nist.gov/~FHunt/appearance/obl.html>

<http://www.graphics.cornell.edu/online/measurements/>

# *Next week...*

- ⇒ More BRDFs
  - Visit microfacet theory in more depth
- ⇒ Assignment #2 due

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